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HEAT AND MASS TRANSFER IN DISPERSE AND POROUS MEDIA

CHARACTERISTIC FEATURES OF TRANSFER PROCESSES IN POLYDISPERSE BEDS ON FULL AND PARTIAL FLUIDIZATION

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Based on the analysis of the processes of full and partial fluidization of a mixture of particles of wide fractional composition, it is shown that the equivalent diameters of particles of a bubbling and fixed beds that form a partially fluidized disperse system constitute hydrodynamic quantities that depend on the velocity of gas percolation and are determined by the Lyashchenko number. An apparatus of the similarity theory of various modifications has been formulated for modeling transport processes in fully and partially fluidized polydisperse beds, including that restrained by a spherical packing.

Keywords: *polydisperse bubbling bed, full and partial fluidization, bed restrained by a spherical packing, full fluidization velocity, equivalent diameter of particles.*

Introduction. Infiltrated beds of polydisperse particles are widely used in practice. Usually, these are fixed fluidized beds and pneumotransport systems. Somewhat unigue is a disperse bed of particles of wide fractional composition in which a regime of partial fluidization is implemented and which actually represents a combination of a fixed bed of larger particles and a bubbling bed of fine fractions. An example of using this system can be the process of burning solid fuel by the technology of a high-temperature circulating bubbling fluidized bed (HTCBFB) [1, 2]. The methods of calculating such polyfractional systems have been developed inadequately as yet. The available recommendations are usually of particular character [3–5], and at the present time there is no single method to take into consideration the polydispersity of particles in describing transport processes in such complex systems. The aim of the present investigation is to justify the recommendations on creating a universal method of calculation of polydisperse granular beds at different regimes of percolation using as a basis the earlier developed similarity theory of transport processes in inhomogeneous fluidized beds [6].

Polydisperse Fluidized Bed of Wide Fractional Composition. At gas percolation velocities $u \geq u_{ff}$ a bed is fully fluidized, and the development of the methods of calculation of the hydrodynamics and heat- and mass transfer is based on the analogy with monodisperse beds, with the equivalent diameter of the particles of a polydisperse mixture having been adequately determined.

Full fluidization velocity. In [3], a formula for calculating the equivalent diameter of the particles of a fluidized bed of agloporit of wide fractional composition has been derived experimentally:

$$d_1^{fb} = \int_{d_t(u)}^{d_{max}} df(d)\, d(d)\,, \tag{1}$$

this diameter determines the velocity of full fluidization u_{ff} from the Todes formula for the minimum fluidization velocity of beds of monofractional particles [4]:

$$u_{ff} = \frac{\nu_f}{d_1^{fb}} \frac{Ar_1}{1400 + 5.22\sqrt{Ar_1}}\,. \tag{2}$$

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In [5], Eq. (1) was generalized to the case of particles of different densities.

Heat and mass transfer. In [5], it was shown that to calculate the external heat transfer one may use the dependences obtained for monodisperse beds if the following quantity is used as the determining diameter of particles:

$$d_2^{\mathrm{fb}} = \left(\int\limits_{d_{\mathrm{t}}(u)}^{d_{\max}} \frac{f(d)}{d}\, d\,(d) \right)^{-1} . \tag{3}$$

The logic of using this quantity is that by its structure the quantity reflects the total surface of particles in a unit volume of the bed. It is evident that precisely such an equivalent diameter must be used in describing the processes proceeding on the surface of particles, that is, the processes of heat and mass transfer.

Similarity between the processes of transfer in polydisperse fluidized beds. In [6], within the framework of the two-phase model of fluidization [7], the similarity theory of transfer processes in monodisperse inhomogeneous fluidized beds has been developed. It is based on the use of the modernized Froude number $\mathrm{Fr}_{\mathrm{mf}} = (u - u_{\mathrm{mf}})^2/(gH_{\mathrm{mf}})$. Its apparatus has the form

$$\begin{array}{ll} \mathrm{Fr}_{\mathrm{mf}},\ \dfrac{H_{\mathrm{mf}}}{D_{\mathrm{a}}},\ \dfrac{h}{H_{\mathrm{mf}}} & \text{– phase of bubbles (hydrodynamics),} \\ \mathrm{Ar}_{\mathrm{d}},\ \dfrac{c_{\mathrm{s}}}{c_{\mathrm{f}}},\ \dfrac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}} & \text{– emulsion phase (hydrodynamics and heat and mass transfer).} \end{array} \tag{4}$$

The coupling between Ar_{d} and $\mathrm{Fr}_{\mathrm{mf}}$ shown by the arrow means that the Archimedes number determines the value of the minimum fluidization velocity that enters into the $\mathrm{Fr}_{\mathrm{mf}}$ number. With allowance for the specificity of a polydisperse bed, where two different equivalent diameters d_1^{fb} and d_2^{fb} appear, generalization of (4) to this case (with the replacement $u_{\mathrm{mf}} \to u_{\mathrm{ff}}$) has the form

$$\begin{array}{ll} \mathrm{Fr}_{\mathrm{ff}},\ \dfrac{H_{\mathrm{ff}}}{D_{\mathrm{a}}},\ \dfrac{h}{H_{\mathrm{ff}}} & \text{– phase of bubbles (hydrodynamics),} \\ \left.\begin{array}{l} \mathrm{Ar}_1 \\ \mathrm{Ar}_2,\ \dfrac{c_{\mathrm{s}}}{c_{\mathrm{f}}},\ \dfrac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}} \end{array}\right\} & \text{emulsion phase} \begin{array}{l} \nearrow \text{hydrodynamics,} \\ \searrow \text{heat and mass transfer.} \end{array} \end{array} \tag{5}$$

Partially Fluidized Polydisperse Bed of Wide Fractional Composition. At percolation velocities $u < u_{\mathrm{ff}}$ a disperse bed has a rather complex structure. In its upper part there is a bubbling bed of fine particles and in the lower part — a fixed bed of large fractions. The process of formation of a partially fluidized bed may follow two paths: either on a decrease in the percolation velocity or on its increase. In the first case the process is independent of the system prehistory, since at $u > u_{\mathrm{ff}}$ the bed is entirely mixed, and on a decrease in the gas velocity a fixed bed is formed as a result of gradual (layer-by- layer) deposition of large fractions on a gas-distribution grid. The process proceeds otherwise on an increase in the gas velocity. In this case, its character depends on the system prehistory. If a fixed bed initially consisted of chaotically arranged particles of different sizes, a bubbling bed is formed as a result of blowing out of fine particles from the fixed bed of large ones that represent a kind of a matrix (the phenomenon of suffosion). However, if a fixed bed is initially structured — the particles can be classified, their layers have been arranged in a rather strict order, and the sizes of particles increase with a decrease in the distance to the gas distributor — a bubbling bed is formed due mainly to the mechanism associated with momentum transfer from the finer suspended particles to the larger ones. We must infer that the processes of formation of a partially fluidized bed as a result of a gradual decrease or increase in the gas velocity are reversible only when a fixed bed was initially structured. In an unstructured system, a bubbling bed is formed due to the action of suffosion. As is known [8], this phenomenon requires a definite relationship between the diameters of the particles of a matrix and of fine fractions, and therefore the disperse composition of a bubbling and fixed beds at identical percolation velocities may differ substantially from one another in different tests and from those that could have taken place in the case of a previously structured bed. Due to this, in what follows we will consider the last variant that yields more reproducible results. For the sake of definiteness it is assumed that a partially fluidized bed was formed in the process of layer-by-layer deposition of large fractions on a gas distributor on a decrease in the percolation velocity from $u_0 = u_{ff}$ to the working one u.

Equivalent diameters of the upper (bubbling) bed. The diameter d_1^{fb} can be calculated from Eq. (2) if the latter is considered as a transcendental equation for d_1^{fb}. An approximate solution of this equation, when u_{ff} is replaced by u, has the form

$$\mathrm{Ar}_1 = (1400 + 742\mathrm{Ly})^{3/2}\,\mathrm{Ly}^{1/2}\,. \tag{6}$$

It can easily be shown that the Reynolds number $\mathrm{Re}_1 = ud_1^{fb}/\nu_f$ can also be expressed in terms of the Lyashchenko number:

$$\mathrm{Re}_1 = \sqrt{(1400 + 742\mathrm{Ly})\,\mathrm{Ly}}\,. \tag{7}$$

Here, an essential fact in the context of the present work should be noted: while in Eq. (2) d_1^{fb} was an argument and u_{ff} was a function, in Eqs. (6) and (7), conversely, the percolation velocity u is an argument and d_1^{fb} is a function. This indicates that the equivalent diameter d_1^{fb} of a partially fluidized bed is in essence a hydrodynamic quantity determined, proceeding from (7), by the percolation velocity as follows

$$d_1^{fb}(\mathrm{Ly}) = \frac{\nu_f}{u}\sqrt{(1400 + 742\mathrm{Ly})\,\mathrm{Ly}}\,. \tag{8}$$

The Lyashchenko number, being a dimensionless flux of the kinetic energy of the gas, plays the role of the Archimedes number for such a system.

At the given general fractional composition of a polydisperse mixture the average diameter d_1^{fb} can be determined as

$$d_1^{fb}(\mathrm{Ly}) = \frac{\int\limits_{d_t(u_{ff})}^{d_{max}^{fb}(u)} df(d)\,d(d)}{\eta^{fb}}\,, \tag{9}$$

where $\eta^{fb} = \int\limits_{d_t(u_{ff})}^{d_{max}^{fb}(u)} f(d)d(d)$ is the fraction of the bubbling bed particles. The levitation diameter $d_t(u_{ff})$ is found from the formula

$$\mathrm{Ar}_t = (18 + 0.14\mathrm{Ly}_{ff})^{3/2}\sqrt{\mathrm{Ly}_{ff}}\,, \tag{10}$$

which is an approximate solution for $d_t(u_{ff})$ of the Todes equation for determining the levitation (terminal) velocity of a single particle [4]:

$$\mathrm{Re}_t = \frac{d_t(u_{ff})\,u_{ff}}{\nu_f} = \frac{\mathrm{Ar}_t}{18 + 0.61\sqrt{\mathrm{Ar}_t}}\,. \tag{11}$$

TABLE 1. Equation (9) for the Two Types of Size Distribution of Particles of the Initial Polydisperse Mixture

Type of initial distribution ($d_t(u_{ff}) < d < d_{max}$)	Distribution density	Eq. (9)
Uniform	$f(d) = \frac{1}{d_{max} - d_t(u_{ff})}$	$d^{fb}_{max}(u) + d_t(u_{ff}) = 2d^{fb}_1 \mathrm{Ly}$
Rosin–Rammler distribution	$f(d) = \frac{n}{a}\left(\frac{d}{a}\right)^{n-1} \exp\left(-\left(\frac{d}{a}\right)^{n}\right), a = \frac{n}{\Gamma\left(\frac{1}{n}\right)} d^{fb}_1(u)$	$a\left[\gamma\left(1+\frac{1}{n}, \left(\frac{d^{fb}_{max}(u)}{a}\right)^{n}\right) - \gamma\left(1+\frac{1}{n}, \left(\frac{d_t(u_{ff})}{a}\right)^{n}\right)\right]$ $= \left[\exp\left(-\left(\frac{d_t(u_{ff})}{a}\right)^{n}\right) - \exp\left(\left(\frac{d^{fb}_{max}(u)}{a}\right)^{n}\right)\right] d^{fb}_1 \mathrm{Ly}$

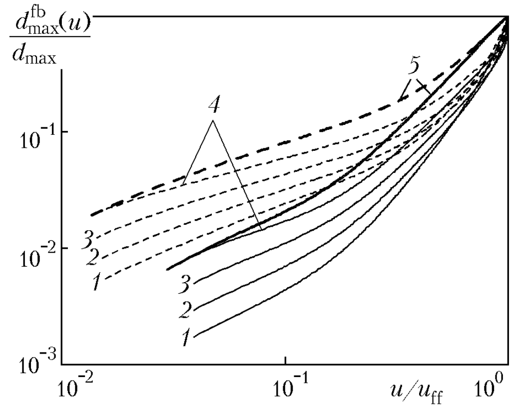


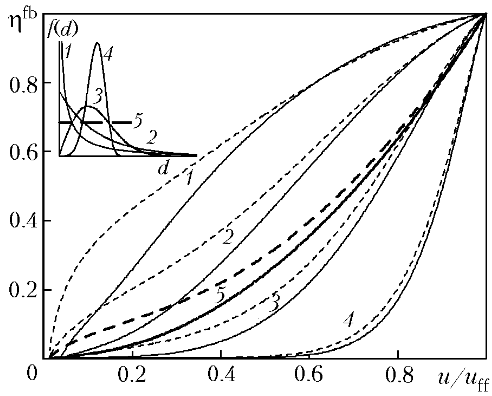
Fig. 1. Dependence of a maximum size of fluidized particles on the percolation velocity: 1–4) the Rosin–Rammler distribution [1) n = 0.5; 2) 1; 3) 2; 4) 5]; 5) uniform distribution; dashed lines, d^{fb}_1 = 0.005 m; solid lines, 0.025 m.

At the given gas velocity u (and, consequently, at the known $d^{fb}_1(\mathrm{Ly})$), relation (9) represents the transcendental equation for the maximum size of fluidized particles $d^{fb}_{max}(u)$. The concrete forms of Eq. (9) for the two types of size distribution function of the particles of a polydisperse mixture are presented in Table 1 and their solutions as functions of the gas velocity are shown in Fig. 1. In the calculation it was assumed that particles with the density ρ_s = 2500 kg/m^3 are fluidized by air under normal conditions. As is seen from the figure, the dependences $d^{fb}_{max}(u)$ represent rather complex monotonic functions with $d^{fb}_{max}(u) \to d_{max}$, when $u \to u_{ff}$. Their character depends on both the type of distribution and its parameters. For the Rosin–Rammler distribution parameters used in the calculation at a known value of d^{fb}_1 the value of d_{max} is defined by the relation

$$\frac{d_{max}}{d^{fb}_1} = \begin{cases} 6n^{-0.7}, & d^{fb}_1 = 0.025 \text{ m};\\ 5n^{-0.6}, & d^{fb}_1 = 0.005 \text{ m}. \end{cases} \quad (12)$$

Knowing $d^{fb}_{max}(u)$, we may calculate the fluidized fraction of the initial polydisperse mixture η^{fb} (Fig. 2). This quantity also represents a very complex monotonic dependence, the character of which is determined by the type of distribution and its parameters. The form of the distribution functions used in the calculation is presented in the inset to Fig. 2. For all the cases the limit $\eta^{fb} \to 1$ for $u \to u_{ff}$ is valid.

It should be noted that Eqs. (6) and (10) can be combined into one universal dependence:



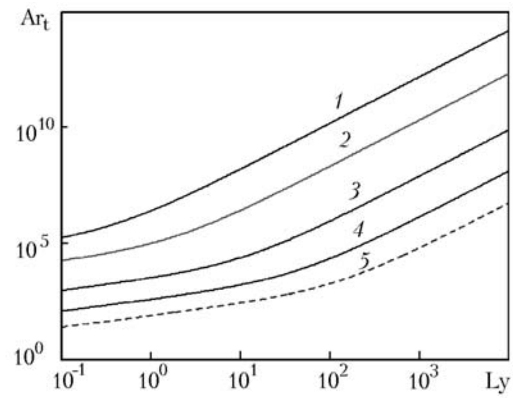


Fig. 2. Fraction of a fluidized material vs. the gas velocity: 1–4) the Rosin–Rammler distribution [1) n = 0.5; 2) 1; 3) 2; 4) 5]; 5) uniform distribution; dashed lines, d_1^{fb} = 0.005 m; solid lines, 0.025 m.

Fig. 3. The Archimedes number vs. the Lyashchenko number at different porosities of the bed: 1) ε = 0.3; 2) 0.4; 3) 0.6; 4) 0.8; 5) 1.

$$\mathrm{Ar_t} = \left(18 + 0.14\,\frac{\mathrm{Ly}}{\varepsilon^{4.75}}\right)^{3/2} \sqrt{\frac{\mathrm{Ly}}{(\varepsilon^{4.75})^3}}\,, \tag{13}$$

that expresses the levitation diameter of a particle under constrained conditions (at ε = 0.4 Eq. (13) is transformed into Eq. (6), and at ε = 1 into Eq. (10)). The family of the curves described by Eq. (13) is shown in Fig. 3.

To calculate the equivalent diameter d_2^{fb} used in describing heat- and mass transfer processes the following formula analogous to Eq. (3) should be used:

$$d_2^{fb}\,(\mathrm{Ly}) = \eta^{fb} \left(\int\limits_{d_t(u_{ff})}^{d_{max}^{fb}(u)} \frac{f(d)}{d}\, d\,(d) \right)^{-1} . \tag{14}$$

Equivalent diameters of the lower (*fixed*) *bed.* First of all, we will determine the average diameter of the particles of the entire partially fluidized bed:

$$\langle d\rangle = \int\limits_{d_t(u_{ff})}^{d_{max}} df\,(d)\; d\,(d) = \int\limits_{d_t(u_{ff})}^{d_{max}^{fb}(u)} df\,(d)\; d\,(d) + \int\limits_{d_{max}^{fb}(u)}^{d_{max}} df\,(d)\; d\,(d)\,. \tag{15}$$

The diameter d_1^{pb} is calculated by the formula that follows from Eqs. (8), (9), and (15):

$$d_1^{pb}\,(\mathrm{Ly}) = \frac{\int\limits_{d_{max}^{fb}(u)}^{d_{max}} df\,(d)\; d\,(d)}{1 - \eta^{fb}} = \frac{\langle d\rangle - \dfrac{\nu_f}{d}\left((1400 + 742\mathrm{Ly})\,\mathrm{Ly}\right)^{1/2} \eta^{fb}}{1 - \eta^{fb}}\,. \tag{16}$$

The diameter d_2^{pb} is determined by the formula analogous to Eq. (14):

$$d_2^{\mathrm{pb}}(\mathrm{Ly}) = (1-\eta^{\mathrm{fb}})\left(\int\limits_{d_{\max}^{\mathrm{fb}}(u)}^{d_{\max}} \frac{f(d)}{d}\, d(d)\right)^{-1}. \tag{17}$$

Hydrodynamics and heat transfer in a partially fluidized bed. The processes of transfer in a bubbling bed which is in the state of a full (minimal) fluidization are calculated according to the recommendations given in the first section without allowance for the phase of bubbles. The resistance of a fixed bed, as well as the processes of heat- and mass transfer, should be calculated from the known [9, 10] dependences for fixed beds of monodisperse particles at $d = d_2^{\mathrm{pb}}$ determined by Eq. (17).

Similarity of transport processes in a partially fluidized bed. With allowance for the established specific features of the system, the apparatus of the similarity theory is formulated as follows:

bubbling bed

$$\begin{aligned} &\mathrm{Ly} && \text{— hydrodynamics}, \\ &\mathrm{Ly},\ \frac{c_{\mathrm{s}}}{c_{\mathrm{f}}},\ \frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{f}}} && \text{— heat and mass transfer}; \end{aligned} \tag{18}$$

fixed bed

$$\langle \mathrm{Re} \rangle,\ \mathrm{Ly} \quad \text{— hydrodynamics and heat and mass transfer}. \tag{19}$$

The occurrence of the $\langle \mathrm{Re} \rangle$ number in Eq. (19) is due to the influence of the average size of particles of the entire bed (see Eq. (16)). From a comparison of Eqs. (5) and (18), (19) one can well see the specificity of the description of a partially fluidized bed: instead of the numbers Ar_1 and Ar_2 for a fully fluidized bed of polydisperse particles in a partially fluidized bed only the Ly number is used that characterizes the lifting force acting on a particle. Moreover (which seems to be one of the specific features of the system), the equivalent diameters d_1^{fb}, d_2^{fb}, d_1^{pb} and d_2^{pb} are unknown quantities of hydrodynamic nature (they depend on the Ly number).

Partially Fluidized Binary Bed of Particles Greatly Differing in Size. The limiting case of a partially fluidized polydisperse system is a blown-through bed consisting of particles of two sizes differing greatly from each other. In a certain range of percolation velocities fine particles may form a bubbling bed in the space between fixed large ones. Fluidization of fine particles in a matrix consisting of large ones is possible at $D_{\mathrm{p}}/d > 10$ (smooth spheres) or $D_{\mathrm{p}}/d > 20$ (particles of irregular shape) [11]. Such a system is called in the literature "a fluidized bed restrained by volume packing," and it is often used in various technical applications, since it ensures a good contact between the gas and particles even at percolation velocities considerably exceeding the minimum fluidization velocity [11, 12].

Hydrodynamics of a bed restrained by a spherical packing. The range of percolation velocities is determined by the following inequalities:

$$\varepsilon_{\mathrm{p}} u_{\mathrm{mf}}^{\mathrm{d}} < u < u_{\mathrm{t}}^{\mathrm{d}}, \quad u < u_{\mathrm{mf}}^{D_{\mathrm{p}}}, \tag{20}$$

where $u_{\mathrm{mf}}^{\mathrm{d}}$, $u_{\mathrm{f}}^{\mathrm{d}}$, and $u_{\mathrm{mf}}^{D_{\mathrm{p}}}$ are determined from the Todes equations (2) and (12) on substitution of diameters with the corresponding superscripts into the Ar number. The region of admissible values of the percolation velocity is shown graphically in Fig. 4 constructed in the coordinates $\mathrm{Ly}_{\mathrm{t}} \to \mathrm{Ar}$ on the basis of the dependence reciprocal of Eq. (13):

$$\mathrm{Ly}_{\mathrm{t}} = \frac{\mathrm{Ar}^2 (\varepsilon^{4.75})^3}{(18 + 0.6\sqrt{\mathrm{Ar}\varepsilon^{4.75}})^3}. \tag{21}$$

The region of values of the Lyashchenko number $[\mathrm{Ly}_{\mathrm{A}}, \mathrm{Ly}_{\mathrm{B}}]$ yields the unknown values of velocities that satisfy inequalities (20).

Provided the above-indicated conditions of fluidization of fine particles in a packing of large ones are satisfied, the character of the fluidization itself depends on the value of D_{p}/d. In [12], experimental data on the depend-

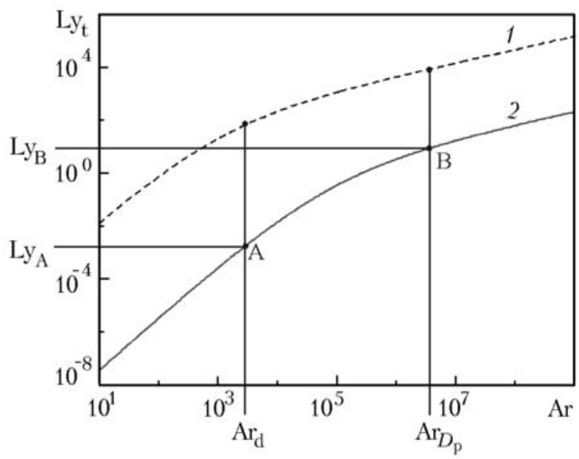


Fig. 4. Terminal velocity of a particle in restrained conditions vs. its diameter: 1) conditions of levitation of a single particle (ε = 1); 2) minimum fluidization (ε = 0.4).

ence of the expansion of a quartz sand, d = 0.23 mm, and corundum, d = 0.125 mm, on the diameter of a spherical packing were obtained (Fig. 5). The extremal form of the dependence of H/H_0 on D_p is well seen. Obviously, this is indicative of the different character of fluidization at small and high values of D_p. The position of the maximum depended on the kind of particles, but in all of the cases it corresponded to the condition $D_p/d \approx 50$. At $D_p/d < 50$, with increasing D_p the expansion of the bed increased, since the mobility of particles increased; at $D_p/d > 50$ the expansion decreased. Judging by the concrete values of H/H_0, we may assume that in the first case fluidization close to homogeneous (quasi-homogeneous) was realized, whereas in the second, inhomogeneous fluidization characterized by the presence of gas bubbles in the bed and, as a consequence, smaller expansion as a result of gas "slippage." It is interesting that in a bed of finer corundum at $D_p/d > 110$ the expansion no longer depends on D_p, and the bed actually behaves as a free one.

Heat and mass transfer. The specifics of the modeling of the processes of transfer in such a system consists in the allowance for the constriction — an increase in the real gas velocity in a packing — as well as for the influence of the simplex D_p/d on the transport properties.

Similarity of transport processes in a fluidized bed restrained by a spherical packing. With allowance for the characteristic features of the hydrodynamics of the system, as well as for the experimental data available in the literature [3, 13], the apparatus of the similarity theory will have the form

$D_p/d < 50$ (quasi-homogeneous fluidization):

$$\mathrm{Ar_d}\,,\ \frac{\mathrm{Re}}{\varepsilon_p}\,,\ \frac{c_s}{c_f}\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D_p}{d}\quad \text{— hydrodynamics and heat and mass transfer}\,; \tag{22}$$

$D_p/d > 50$ (inhomogeneous fluidization):

$$\begin{gathered}\mathrm{Ar_d}\,,\ \mathrm{Fr}^{p}_{mf} = \frac{\left(\dfrac{u}{\varepsilon_p} - u_{mf}\right)^2}{gD_p}\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D_p}{d}\quad \text{— phase of bubbles (hydrodynamics)}\,,\\ \mathrm{Ar_d}\,,\ \frac{c_s}{c_f}\,,\ \frac{\rho_s}{\rho_f}\,,\ \frac{D_p}{d}\quad \text{— emulsion phase (hydrodynamics and heat and mass transfer)}\,.\end{gathered} \tag{23}$$

As is seen, in contrast to system (4), in Eqs. (22) and (23), in addition to D_p/d, use is made also of $\mathrm{Ar_d}$ and ρ_s/ρ_f in describing hydrodynamic processes connected with the presence of gas bubbles. This is explained by the

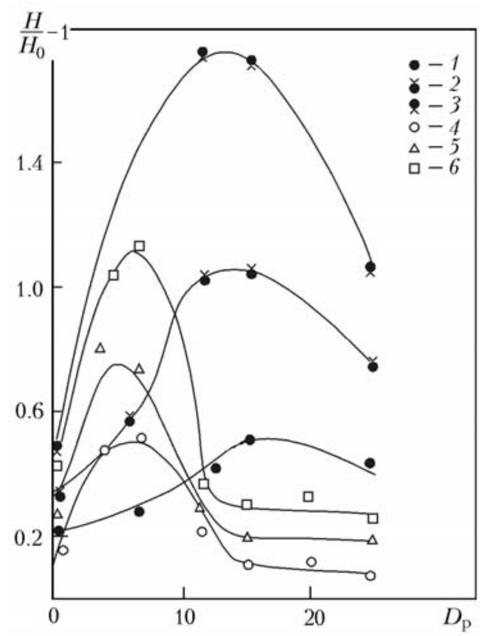


Fig. 5. Dependence of the relative expansion of a fluidized bed on the diameter of an element of a spherical packing: 1–3) quartz sand with fluidization numbers 2, 4, and 6, respectively; 4–6) corundum with fluidization numbers 2, 4, and 6, respectively. D_p, mm.

restraining influence of packing on the bubble phase (the size and velocity of the rise of gas bubbles). The availability of these additional similarity numbers has been confirmed experimentally in experiments on expansion of beds in low-volume [3] and spherical [13] packings.

Conclusions. For the first time, from unified methodological standpoints the processes of transport in an infiltrated disperse system of wide fractional composition consisting of bubbling and fixed (partially fluidized) beds have been considered. It is shown that the principal distinctive feature of such a system is the hydrodynamic character of the equivalent diameters of particles that are determined by the Lyashchenko number. The apparatus of the similarity theory of transport processes in a partially fluidized bed has been formulated, which represents a new form in an infiltrated disperse system.

NOTATION

$$\mathrm{Ar}_1 = \frac{g(d_1^{\mathrm{fb}})^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f}-1\right),\ \mathrm{Ar}_2 = \frac{g(d_2^{\mathrm{fb}})^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f}-1\right),\ \mathrm{Ar_d} = \frac{gd^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f}-1\right),\ \mathrm{Ar}_{D_p} = \frac{g(D_p)^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f}-1\right),\ \mathrm{Ar_t} = \frac{gd_t^3}{\nu_f^2}\left(\frac{\rho_s}{\rho_f}-1\right),$$

the Archimedes numbers; c_f, c_s, heat capacities of a gas and particles, J/(kg·K); d, diameter of a particle, m; d_t, diameter of particles entrained at terminal velocity, m; D_a, diameter of the apparatus, m; D_p, diameter of particles of spherical packing, m; $\mathrm{Fr_{mf}} = \frac{(u-u_{mf})^2}{gH_{mf}}$, $\mathrm{Fr_{ff}} = \frac{(u-u_{ff})^2}{gH_{ff}}$, $\mathrm{Fr_{mf}^p} = \frac{(u/\varepsilon_p - u_{mf})^2}{gD_p}$, the Froude numbers; $f(d)$, distribution density, 1/m; g, free fall acceleration, m/sec^2; H, height of a disperse bed, m; h, current height, m; $\mathrm{Ly} = \frac{\mathrm{Re}^3}{\mathrm{Ar_d}} = \frac{\rho_f u^3}{(\rho_s-\rho_f)\nu_f g}$, $\mathrm{Ly_{ff}} = \frac{\rho_f u_{ff}^3}{(\rho_s-\rho_f)\nu_f g}$, $\mathrm{Ly_t} = \frac{\rho_f u_t^3}{(\rho_s-\rho_f)\nu_f g}$, the Lyashchenko numbers; n, the Rosin–Rammler distribution parameter; $\mathrm{Re} = ud/\nu_f$, $\mathrm{Re_t} = ud_1/\nu_f^{\mathrm{fb}}$; $\mathrm{Re_t} = ud_t/\nu_f$, $\langle\mathrm{Re}\rangle = u\langle d\rangle/\nu_f$, the Reynolds numbers; u, percolation velocity, m/sec; γ, incomplete gamma function; Γ, gamma function; ε_p, porosity of a packing; ε, porosity of a bed; ν_f, kinematic viscosity of a gas, m^2/sec; ρ_f, ρ_s, density of a gas and particles, kg/m^3. Indices: d, fluidized particles; fb, fluidized bed; pb, packed (fixed) bed; p, packing; a, apparatus; f, fluid (gas); ff, full fluidization; mf, minimum fluidization; max, maximum; s, particles; t, terminal; 0, initial.

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